A

16131 **120 MINUTES**

1. In the set R of real numbers, the functions f and g are defined as the greatest integer function and the modulus function respectively. If o represents the composition between two functions, then the value of
$(g \circ f)\left(\frac{5}{3}\right) - (f \circ g)\left(\frac{4}{3}\right)$ is equal to

A) 1 B) 0 C) $\frac{4}{3}$ D) $\frac{5}{3}$

2. The sum of cubes of the first n positive integers is equal to

A) $\frac{n(n+1)^2}{4}$ B) $\frac{n^2(n+1)}{4}$

C) $\left\{\frac{n(n+1)}{2}\right\}^2$ D) $\frac{n(n+1)^2\,(2n+1)}{6}$

3. Given that $\{a_n\}$ is a non-increasing sequence of positive numbers such that $\sum_{n=1}^{\infty} a_n < \infty$. Which of the following is now true?

A) $\lim_{n\to\infty} na_n = 0$ B) $\lim_{n\to\infty} na_n = 1$

C) $\lim_{n\to\infty} na_n = +\infty$ D) $\lim_{n\to\infty} na_n > 0$

4. Consider the function:

$$f(x) = \begin{cases} -1, \text{ if } x < -1 \\ -x, \ -1 \le x \le 1 \\ 1, \ x > 1 \end{cases}$$

Now state which of the following is correct?
A) f is continuous at both of the points x = 1 and x = –1
B) f is continuous at x = +1 but not at x = –1
C) f is continuous at x = –1but not at x = +1
D) f is not continuous at the points x = +1 and x = –1

5. A real number in the interval [0,1] denoted by x is said to have a ternary expansion with digits 0, 1 and 2, if we can write $x = \sum_{i=1}^{\infty} \frac{bi}{3^i}$, where bi $\in \{0, 1, 2\}$, i = 1, 2,… or by x = $0.b_1b_2b_3$ … Now state which of the following can be taken as the ternary expansion of the usually known rational number ½?

A) 0.1111 … B) 0.2222 … C) 0.5000 … D) 0.1212 …

6. Let N, I, Q and R be the set of natural numbers, the unit interval [0,1], the set of positive rationals and the set of real numbers respectively. If we write X^n to denote the cartesian product of n copies of the set X and consider the statements: (i) N^n is countable (ii) I^n is countable (iii) Q^n is countable (iv) R^n is countable then state which of the following is correct?

A) (i) only
B) (i) and (ii) only
C) (i), (ii) and (iii) only
D) (i), (iii) and (iv) only

7. Given A is nonempty open set on the real line R with A^0 the set of interior points of A. Now consider the following statements:
(i) A can be obtained as the union of a countable disjoint class of open intervals.
(ii) A^0 is a open set
(iii) $A^0 = A$
State which of the following is correct?

A) Only (ii) is true
B) Only (iii) is true
C) Only (ii) and (iii) are true
D) (i), (ii), and (iii) are all true

8. Consider the interval [0,1) of R. Then the limit points of the interval are given by the set:
A) {1}
B) {0,1}
C) [0,1]
D) [0,1)

9. If C is the circle given by $|z| = 2$, then the value of the integral $\int_c \frac{dz}{z^2+1}$ is equal to

A) 0
B) $\frac{\Pi}{2i}$
C) 4Π
D) $-\frac{\Pi}{2i}$

10. If $f(x + iy) = x^2 + y^2$, then which of the following is correct?
A) f is differentiable at z = i only
B) f is differentiable at z = – i only
C) f is differentiable at z = 0 only
D) f is differentiable at all z.

11. The value of the line integral $\int_0^{1+i} z^2 dz$ is
A) $-1 + i$
B) $1 - i$
C) $\frac{-2}{3} + \frac{2}{3}i$
D) $\frac{2}{3} + \frac{2}{3}i$

12. If V is a vector space and $W \subset V$ is a subspace, define for $u \in V$, $u + W = \{v = u+w \mid w \in W\}$. For $\alpha \in F$, where F is the field of scalars, consider the following statements:
(i) $u + W$ is a subspace of V for all $u \in V$
(ii) If $\alpha W = \{\alpha w \mid w \in W\}$ it follows that $\alpha W = W$
(iii) $\alpha(u+W) = \alpha u + W$
Now state which of the following is correct?

A) (i) only
B) (i) and (ii) only
C) (i) and (iii) only
D) (ii) and (iii) only

13. Given V is a vector space and T is a linear transformation on V. Then which of the following is not correct?

A) If 0 is the additive identity of V then $\{u \in V \mid u\,T = 0\}$ is a subspace of V.
B) If the range of T is defined by $VT = \{uT \mid u \in V\}$, then VT is not a subspace of V
C) $(u_1 + u_2)\,T = u_1T + u_2T$ for all $u_1, u_2 \in V$
D) $(\alpha u)T = \alpha(uT)$ for all $u \in V$ and $\alpha \in F$ where F is the field of scalars.

14. Given U and W are subspaces of a vector space V. Consider the statements:
(i) $U \cup W$ is a subspace of V
(ii) $U \cap W$ is a subspace of V
(iii) $U + W = \{v \in V \mid v = u + w, u \in U, w \in W\}$ is a subspace of V.
Now state which of the following is true?

A) (i) and (ii) only
B) (i) and (iii) only
C) (ii) and (iii) only
D) All of (i), (ii) and (iii)

15. Which of the following is not true concerning the eigen values of a matrix A of order n x n?
A) The eigen values of an orthogonal matrix are ± 1
B) If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the eigen values of A, then trace (A) = $\sum_{i=1}^{n} \lambda_i$
C) If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the eigen values of A, then $|A| = \prod_{i=1}^{n} \lambda_i$
D) If the eigen values of a matrix A are 0's and 1's, then A is idempotent

16. If $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$, then state which of the following is correct?
A) $A^{-1} = A^2 - 5A + 7I$
B) $A^{-1} = -A^2 + 5A - 7I$
C) $A^{-1} = \frac{1}{3}A^2 - \frac{5}{3}A + \frac{7}{3}I$
D) $A^{-1} = -\frac{1}{3}A^2 + \frac{5}{3}A - \frac{7}{3}I$

17. Given $\{E_n\}$ is a sequence of intervals of R defined by:

$$E_n \begin{cases} (-n-1, -n+1], \text{ if n is odd} \\ (n-1, n+1], \text{ if n is even} \end{cases}$$

Then which of the following is equal to $\lim\limits_{n \to \infty} E_n$?

A) $(-\infty, 0]$ B) Empty set C) $(0, \infty)$ D) $(-\infty, \infty)$

18. Consider the following sequence of functions:

$$f_n(x) = \begin{cases} n^{1/p}, x\varepsilon\left(0, \frac{1}{n}\right] \\ 0, \text{otherwise} \end{cases}$$

and the statements:

(i) $f_n \to 0$ almost uniformly but $f_n \nrightarrow 0$ in p^{th} mean

(ii) $f_n \to 0$ in measure but $f_n \nrightarrow 0$ in p^{th} mean

(iii) $f_n \to 0$ pointwise as well as in p^{th} mean

Now state which of the following is correct?

A) (i) only B) (ii) only

C) (i) and (ii) only D) all of (i), (ii) and (iii)

19. Given g, h and f_n, n = 1, 2, … are non-negative μ measurable functions such that g (x) ≤ $f_n(x)$ ≤ h(x) for all x ε X and n = 1, 2, …. Now consider the following statements:

(i) $\int \liminf\limits_{n\to\infty} f_n d\mu \leq \liminf\limits_{n\to\infty} \int f_n d\mu$

(ii) $\int \liminf\limits_{n\to\infty} f_n d\mu \geq \liminf\limits_{n\to\infty} \int f_n d\mu$

(iii) $\int \limsup\limits_{n\to\infty} f_n d\mu \leq \limsup\limits_{n\to\infty} \int f_n d\mu$

(iv) $\int \limsup\limits_{n\to\infty} f_n d\mu \geq \limsup\limits_{n\to\infty} \int f_n d\mu$

Which of the following is then correct?

A) (i) and (iii) B) (i) and (iv)

C) (ii) and (iv) D) (ii) and (iii)

20. $(X,\boldsymbol{B}\mu)$ is a measure space, λ_1 and λ_2 are also measures defined on $(X,\mathbf{B})$ such that $\lambda_1 << \mu$ and $\lambda_2 \perp \mu$. Then which of the following is correct?

A) $\lambda_1 << \lambda_2$ B) $\lambda_2 << \lambda_1$ C) $\lambda_1 \perp \lambda_2$ D) $\lambda_1 \perp \mu$

21. Given $F(x) = \dfrac{1}{1+e^{-x}}, -\infty < x < \infty$ is a Stieltjes measure function. Then the Lebesgue – Stietljes measure of the interval (–1, 1) is equal to:

A) 2 B) $\dfrac{e-1}{e+1}$ C) $\dfrac{1}{1+e^{-1}}$ D) $\dfrac{1}{1+e}$

22. Given the events A and B with $P(A) = 0.4$, $P(B) = p$ and $P(AUB) = 0.8$. Then the value of p for which A and B are independent is

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) $\frac{2}{15}$ D) $\frac{2}{5}$

23. An urn contains ten balls of which three are black and seven are white. A play consists of selecting at each draw a ball, its colour noted and two additional balls of the same colour replaced in the urn. What is the probability that a black ball is drawn in each of the first three draws?

A) $\frac{1}{14}$ B) $\frac{7}{14}$ C) $\frac{1}{16}$ D) $\frac{15}{392}$

24. Consider an experiment of throwing two unbiased dice. A_1 denotes the event of an odd face on the first die, A_2, the event of an odd face on the second die and A_3, the event of an odd total in the experiment. Observe the statements:

(i) A_1, A_2 and A_3 are mutually independent
(ii) A_1 and A_2 are independent
(iii) A_1 and A_3 are independent
(iv) A_2 and A_3 are independent

Now state which of the following is correct?

A) (i), (ii), (iii) and (iv) B) (ii) and (iii) only
C) (i) and (iii) only D) (ii), (iii) and (iv) only

25. Let X be a random variable and $E|X|^k < \infty$ for $k > 0$. Then $\lim_{n\to\infty} n^k P\{|X| > n\}$ is equal to

A) 0 B) 1 C) $\frac{1}{n^k}$ D) $\frac{1}{n}$

26. Let X be a non-negative but integer valued type random variable and $P(X > r + t / X > t) = P(X \geq r)$ for positive integers r and t. Then the distribution of X is

A) geometric B) binomial
C) Poisson D) negative binomial

27. The probability generating function of geometric distribution is

A) $(q + ps)^n$ B) $p(1 - qs)^{-1}$
C) $qe^{p(s-1)}$ D) $\frac{1-(q+ps)^n}{1-s}$

28. Identify the distribution for which the characteristic function is given by $\phi(t) = \exp[i\ t - \theta|t|]$ from among the following

A) Exponential distribution B) Beta distribution
C) Cauchy distribution D) Laplace distribution

29. Let X be a random variable with E(X) = 0 and variance equal to σ^2 .
Consider the statements:

(i) $P(X > x) \leq \frac{\sigma^2}{\sigma^2 + x^2}$, if $x > 0$ (ii) $P(X > x) \geq \frac{\sigma^2}{\sigma^2 + x^2}$, if $x > 0$

iii) $P(X > x) \leq \frac{\sigma^2}{\sigma^2 + x^2}$, if $x < 0$ (iv) $P(X > x) \geq \frac{\sigma^2}{\sigma^2 + x^2}$, if $x < 0$

Now state which of the following is correct?

A) (i) & (iv) B) (i) &(iii) C) (ii) & (iii) D) (ii) & (iv)

30. For a sequence $\{X_n\}$ of iid Cauchy random variables, consider the statements:
(i) Kolmogorov strong law of large numbers holds
(ii) Lindeberg – Levy central limit theorem holds
Now state which of the following is correct?

A) Only (i) is true B) Only (ii) is true
C) Both (i) and (ii) are true D) Both (i) and (iii) are not true

31. For a sequence $\{X_n\}$ of iid random variables, which of the following is true?
A) Convergence in probability $\Rightarrow$ almost sure convergence
B) Convergence in probability $\Rightarrow$ convergence in pth mean
C) Convergence in pth mean $\Rightarrow$ almost sure convergence
D) If the given sequence of random variables are both non-negative and strictly decreasing such that $X_n \xrightarrow{P} 0$ then $X_n \xrightarrow{a.s} 0$.

32. The joint probability density function of a bivariate random variable (X,Y) is given by

$f(x,y) = x + y,\ 0 < x < 1,\ 0 < y < 1$

Which of the following is the conditional pdf of Y given X= x?

A) $\frac{x+y}{x+\frac{1}{2}}$, $0 < y < 1$ B) $\frac{2xy+y}{x+\frac{1}{2}}$, $0 < y < 1$

C) $\frac{x+\frac{3y^2}{2}}{x+\frac{1}{2}}$, $0 < y < 1$ D) $\frac{2xy+\frac{3y^2}{2}}{x+\frac{1}{2}}$, $0 < y < 1$

33. X is a binomial random variable and its distribution is B (6, p). If
P (X = 1) = P(X = 2), then the mean and variance σ^2 of the distribution are given by

A) $= 6p,\ \sigma^2 = 6pq$ B) $= 3,\ \sigma^2 = \frac{3}{2}$

C) $= \frac{12}{7},\ \sigma^2 = \frac{60}{49}$ D) $= \frac{2}{7},\ \sigma^2 = \frac{10}{49}$

34. X is a Poisson random variable and it is given that $P(X = 1) = P(X = 3)$. Then the variance of the random variable X is equal to

A) 6 B) 4 C) 2 D) $\sqrt{6}$

35. Given X_1 and X_2 are independent random variables each distributed identically as that of the random variable X. Further it is known that $X_1 + X_2$ and $|X_1 - X_2|$ are independently distributed. Then the distribution of X is

A) Gamma B) Normal C) Chi-square D) Cauchy

36. If the random variable X is distributed as standard Laplace, then the distribution of the random variable Y defined by $Y = \begin{cases} \frac{1}{2} e^{X}, \text{ for } X \le 0 \\ 1 - \frac{1}{2} e^{-X}, \text{ for } X \ge 0 \end{cases}$ is given as:

A) Exponential B) Normal
C) Uniform D) Double exponential

37. Given X_1, X_2, X_3, X_4 are the observations of a random sample of size 4 from a distribution with cumulative distribution function F(x). Now identify the statistic for which the distribution function is $4[F(x)]^3 - 3\,[F(x)]^4$ from among the following

A) First order statistic B) Second order statistic
C) Third order statistic D) Largest observation in the sample

38. If F(x) is the distribution function of a random variable X, then state which of the following is not a distribution function?

A) $[F(x)]^5$ B) $1 - [1 - F(x)]^5$
C) $5\,[F(x)]^5 - 4\,[F(x)]^4$ D) $5\,[F(x)]^4 - 4\,[F(x)]^5$

39. For the Laplace distribution with pdf

$$f(x,\ \ ,\sigma) = \frac{1}{2\sigma}\, e^{-\frac{|x - \ \ |}{\sigma}}$$

the quartile deviation is equal to

A) $\frac{2}{3}\sigma$ B) $\sigma \log_e 2$ C) $2\sigma \log_e 2$ D) $\sqrt{\frac{2}{\pi}}\,\sigma$

40. The joint pdf of random variables X and Y is given by

$f(x, y) = \frac{1}{\sqrt{2\pi}} e^{\frac{-1}{2}(y-1-2x)^2}$, $-\infty < y < \infty$, $0 < x < 1$ and 0 elsewhere. Then the marginal distribution of X is given by the pdf:

A) $f_1(x) = \begin{cases} 1, 0 < x \leq 1 \\ 0, \text{otherwise} \end{cases}$ B) $f_1(x) = \begin{cases} 2x, 0 \leq x \leq 1 \\ 0, \text{otherwise} \end{cases}$

C) $f_1(x) = \begin{cases} 3x^2, 0 \leq x \leq 1 \\ 0, \text{otherwise} \end{cases}$ D) $f_1(x) = \begin{cases} 4x^3, 0 \leq x \leq 1 \\ 0, \text{otherwise} \end{cases}$

41. For the bivariate distribution defined in question No.40, E(Y) is equal to

A) $1 + 2x$ B) 0
C) 1 D) 2

42. A trivariate (X_1, X_2, X_3) random variable has the following discrete probability function:

Sample point (x_1, x_2, x_3)	(0,0,0)	(0,0,1)	(0,1,1)	(1,0,1)	(1,1,0)	(1,1,1)
Probability	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

Consider the statements:
(i) X_1 and X_2 are identically distributed but not independent
(ii) X_1 and X_3 are identically distributed as well as independently distributed
(iii) X_2 and X_3 are neither identically nor independently distributed
Now state which of the following is correct?

A) (i) only B) (iii) only
C) (ii) only D) (i) and (iii) only

43. Let X_m be the median of a random sample of size n from N ($,\sigma^2$). Then the asymptotic distribution of X_m is

A) $N\left(\;, \frac{\sigma^2}{n} \right)$

B) One with pdf $f_{x_m}(x) = \frac{1}{\sigma} \exp\left\{ \frac{x - \;}{\sigma} - e^{-\frac{x-\;}{\sigma}} \right\}$

C) $N\left(\;, \frac{\pi\sigma^2}{2n} \right)$

D) $N\left(\;, \frac{\pi\sigma^2}{n} \right)$

44. Suppose we have a four variate normal population and the simple correlation between ith and jth variables is ρ for all $i \neq j = 1, 2, 3, 4$. Then the partial correlation coefficient of order two is equal to

A) $\dfrac{\rho}{1+2\rho}$ B) $\dfrac{1}{1+\rho}$ C) $\dfrac{\rho^2}{1+2\rho^2}$ D) $\dfrac{\rho}{1+4\rho}$

45. Let $X_1, X_2, X_3, X_4, X_5, X_6, X_7$ be seven independent observations drawn from $N(0, \sigma^2)$. Then which of the following has a student's t-distribution with four degrees of freedom?

A) $\dfrac{X_1 + X_2 + X_3}{\sqrt{X_4^2 + X_5^2 + X_6^2 + X_7^2}}$ B) $\dfrac{2(X_1 + X_2 + X_3)}{\sqrt{X_4^2 + X_5^2 + X_6^2 + X_7^2}}$

C) $\dfrac{X_1 + X_2 + X_3}{\sqrt{3(X_4^2 + X_5^2 + X_6^2 + X_7^2)}}$ D) $\dfrac{2(X_1 + X_2 + X_3)}{\sqrt{3(X_4^2 + X_5^2 + X_6^2 + X_7^2)}}$

46. If $X_1, X_2, X_3, X_4, X_5, X_6, X_7$ are iid $N(0, \sigma^2)$ variables, which of the following has an F distribution with (3,4) degrees of freedom?

A) $\dfrac{X_1^2 + X_2^2 + X_3^2}{\sum_{i=1}^{7} X_i^2}$ B) $\dfrac{X_1^2 + X_2^2 + X_3^2}{\sum_{i=4}^{7} X_i^2}$

C) $\left\{\dfrac{4\sum_{i=1}^{3} X_i^2}{3\sum_{i=4}^{7} X_i^2}\right\}^{1/2}$ D) $\dfrac{4\sum_{i=1}^{3} X_i^2}{3\sum_{i=4}^{7} X_i^2}$

47. Let X be a p-variate random vector having a multivariate normal distribution with mean vector and variance-covariance matrix Σ. If $t' = (t_1, t_2, .., t_p)$ is a vector of parameters, then which of the following represents the mgf, $M_x(t) = E(e^{t'x})$?

A) $e^{t' \quad + \frac{1}{2}t'\Sigma t}$ B) $e^{t' \quad - \frac{1}{2}t'\Sigma t}$

C) $e^{-t' \quad + \frac{1}{2}t'\Sigma t}$ D) $e^{t' \quad + t'\Sigma t}$

48. A function F(x,y) is a bivariate distribution function. Then which of the following should be satisfied by F(x,y) on the vertices of the rectangular sets $\{(x,y): x_1 < x \leq x_2, y_1 < y \leq y_2\}$?

A) $F(x_1,y_1) - F(x_1,y_2) - F(x_2,y_1) + F(x_2,y_2) \geq 0$
B) $- F(x_1,y_1) + F(x_1,y_2) + F(x_2,y_1) - F(x_2,y_2) \geq 0$
C) $- F(x_1,y_1) - F(x_1,y_2) + F(x_2,y_1) + F(x_2,y_2) \geq 0$
D) $F(x_1,y_1) + F(x_1,y_2) - F(x_2,y_1) - F(x_2,y_2) \geq 0$

49. Let $\overline{X}$ be the mean and M the median of a random sample of size n drawn from N($,\sigma^2$) with an information that μ ε (a,b) where a and b are known reals. Let T be another statistic defined by

$$T = \begin{cases} a, \text{if } \overline{X} < a \\ \overline{X}, \text{if } a \le \overline{X} \le b \\ b, \text{if } \overline{X} > b \end{cases}$$

Now state which of the following is correct?

A) $\overline{X}$ is the most preferred estimator of
B) T is the most preferred estimator of
C) M is the most preferred estimator of
D) $\overline{X}$, T and M are all equally preferred estimators of

50. If $X_1, X_2, \ldots, X_n$, is a random sample of size n drawn from the beta distribution with pdf

$$f(x;a,b) = \begin{cases} \dfrac{1}{B(a,b)} x^{a-1} (1-x)^{b-1}, 0 < x < 1 \\ 0 \qquad\qquad , \quad \text{otherwise} \end{cases}$$

Then, state which of the following is minimal sufficient statistic for the above family of distributions:

A) $\left(\sum_{i=1}^{n} \log X_i, \sum_{i=1}^{n} \log(1-X_i)\right)$ B) $\left(\sum_{i=1}^{n} X_i, n - \sum_{i=1}^{n} X_i\right)$

C) $\sum_{i=1}^{n} X_i$ D) $n - \sum_{i=1}^{n} X_i$

51. If $X_1, X_2, \ldots, X_n$, is a random sample of size n drawn from the Poisson distribution with pmf $f(x;\theta) = \dfrac{e^{-\theta}\theta^x}{x!}, x = 0, 1, 2, \ldots$, then the Cramer – Rao lower bound for the variance of any unbiased estimator for $e^{-\theta}$ is

A) $\dfrac{e^{-2\theta}}{n}$ B) $\dfrac{\theta e^{-2\theta}}{n}$ C) $\dfrac{\theta}{n}$ D) $\dfrac{e^{-\theta}}{n}$

52. Let $X_1, X_2, \ldots, X_n$, be a random sample of size n drawn from N($,\sigma^2$). Define

$$T = \frac{1}{\sqrt{2}} \left\{\sum_{i=1}^{n} (X_i - \overline{X})^2\right\}^{1/2} \left\{\frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\frac{n}{2}}\right\}^{1/2}$$

. Consider the statements given below:

(i) T is an unbiased estimator of σ (ii) T is a consistent estimator of σ
(iii) T is the UMVUE of σ. Now state which of the following is correct?

A) Only (i) is true B) Only (ii) is true
C) Only (i) and (ii) are true D) All (i), (ii) and (iii) are true

53. If $X_{1:n}$ and $X_{n:n}$ are the 1st and nth order statistics of a sample of size n drawn from the uniform distribution U $(\theta, 2\theta)$, then state which of the following is correct?

A) $X_{1:n}$ is the minimal sufficient statistic for θ
B) $X_{n:n}$ is the minimal sufficient statistic for θ
C) $(X_{1:n}, X_{n:n})$ is the minimal sufficient statistic for θ
D) The mean $\overline{X}$ is the minimal sufficient statistic for θ

54. Consider the Cauchy distribution defined by the pdf $f(x;\theta) = \frac{1}{\pi}\frac{1}{1+(x-\ )^2}, -\infty < x < \infty, -\infty < \ \ < \infty$ and the following statements:

(i) $f(x,\theta)$ belongs to the exponential family of distributions
(ii) $f(x,\theta)$ belongs to the Pitman family of distributions
Now state which of the following is correct?

A) Only (i) is true
B) Only (ii) is true
C) Both (i) and (ii) are true
D) Neither (i) nor (ii) is true

55. Life times of n units each distributed with pdf $f(x,\theta) = \theta e^{-\theta x}$, $x > 0$ are to be observed. In this experiment, the life time of units are recorded if they do not exceed a pre-assigned value A. Suppose there are n-m units whose life times exceed A and the exact life times of other units are $X_1, X_2, \ldots, X_m$, then the maximum likelihood estimators of θ is

A) $\dfrac{\sum_{j=1}^{m} X_j}{m}$
B) $\left\{\sum_{j=1}^{m} \dfrac{X_j}{m}\right\}^{-1}$
C) $\dfrac{m}{\sum_{j=1}^{m} X_j + A(n-m)}$
D) $\dfrac{1}{\frac{1}{m}\sum_{j=1}^{m} X_j + A}$

56. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n drawn from $N(\ ,\sigma^2)$ with both μ and σ^2 unknown. Define $s^2 = \sum_{i=1}^{n} \dfrac{(X_i - \overline{X})^2}{n-1}$ and q_2 and q_1 as the upper $(\alpha/2)^{th}$ quantile and lower $(\alpha/2)^{th}$ quantile respectively of a chi-square distribution with n-1 degrees of freedom. Then the $(1-\alpha)100\%$ confidence interval for σ^2 is

A) $\left(\dfrac{(n-1)s^2}{q_2}, \dfrac{(n-1)s^2}{q_1}\right)$
B) $\left(\dfrac{ns^2}{q_2}, \dfrac{ns^2}{q_1}\right)$
C) $\left(\dfrac{q_1}{(n-1)s^2}, \dfrac{q_2}{(n-1)s^2}\right)$
D) $\left(\dfrac{(n-1)s^2}{q_1}, \dfrac{(n-1)s^2}{q_2}\right)$

57. X and Y are two independent observations drawn from the uniform distribution U $(0,\theta)$. If the test function proposed to test H_0: $\theta=1$ against $H_1 : \theta = 2$ is $\phi(X,Y) = (X+Y)/4$, then the size of the test α and power of the test p are given by

A) $\alpha = 0.25, p = 0.5$ B) $\alpha = 0.5, p = 0.75$
C) $\alpha = 0.5, p = 0.8$ D) $\alpha = 0.25, p = 0.75$

58. Consider the following statements:
(i) A consistent estimate provides a consistent test
ii) Under certain conditions a likelihood ratio test is a consistent test
iii) With respect to every consistent test there is a consistent estimate
Now state which of the following is correct?

A) (ii) only B) (i) and (ii) only
C) (ii) and (iii) only D) All of (i), (ii) and (iii)

59. Let $X_{i_1}, X_{i_2}, \ldots, X_{i_{n_i}}$ be a random sample of size n_i drawn from N($_i$,σ_i^2) for i = 1,2. Then for known $_1$ and $_2$ which of the following statistics cannot be used for an F-test for testing $H_0:\sigma_1^2 = \sigma_2^2$?

A) $$\frac{\frac{1}{n_1}\sum_{j=1}^{n_1}(X_{1j} - \ _1)^2}{\frac{1}{n_2}\sum_{j=1}^{n_2}(X_{2j} - \ _2)^2}$$

B) $$\frac{\frac{1}{n_1}\sum_{j=1}^{n_1}(X_{1j} - \bar{X}_1)^2}{\frac{1}{n_2}\sum_{j=1}^{n_2}(X_{2j} - \bar{X}_2)^2}$$

C) $$\frac{\frac{1}{n_2}\sum_{j=1}^{n_2}(X_{2j} - \ _2)^2}{\frac{1}{n_1}\sum_{j=1}^{n_1}(X_{1j} - \ _1)^2}$$

D) $$\frac{\frac{1}{n_1-1}\sum_{j=1}^{n_1}(X_{1j} - \bar{X}_1)^2}{\frac{1}{n_2-1}\sum_{j=1}^{n_2}(X_{2j} - \bar{X}_2)^2}$$

60. The sample correlation coefficient of 83 observations drawn from a bivariate normal population is 0.8. In this case what is the value of the Student's t-statistic for testing the hypothesis that the population correlation coefficient is equal to zero?

A) 12 B) $\frac{27}{4}$ C) 36 D) $\frac{9}{4}$

61. Suppose $\{X_i\}$ is a sequence of iid random variables with a common distribution $N(1,\sigma^2)$. Define $S_N = \sum_{i=1}^{N} X_i$ where N is an integer valued random variable independent of X_i, i =1,2, … and the pmf is defined by $P(N=n) = pq^{n-1}$, n= 1,2,…, $0 < p < 1$ and $q = 1 - p$. Then the value of $E(S_N)$ is equal to

A) q B) $\frac{N}{p}$ C) $\frac{1}{p}$ D) Np

62. Suppose $X_{n_1:n}$ and $X_{n_2:n}$ are the n_1^{th} and n_2^{th} order statistics of a random sample of size n (for $1 \leq n_1 < n_2 \leq n$) drawn from an absolutely continuous cdf F(x). Let $\xi_{1/2}$ be the median of F(x). Then $\left(P(X_{n_1:n} \leq \xi_{\frac{1}{2}} \leq X_{n_2:n} \right)$ is equal to

A) $\int_{X_{n_1:n}}^{X_{n_2:n}} dF(x)$

B) $\int_{X_{n_1:n}}^{X_{n_2:n}} f_m(x)\,dx$, where $f_m(x)$ is the pdf of the sample median

C) $F_m(X_{n_2:n}) - F_m(X_{n_1:n})$, where $F_m(x)$ is the cdf of the sample median

D) $\sum_{i=n_1}^{n_2} \binom{n}{i} \left(\frac{1}{2}\right)^n$

63. Suppose 2.3, 1.8, 0.7, 1.0, 2.9, 3.2, 1.7 and 1.6, 1.2, 2.7, 0.9, 1.1, 2.4, 2.8 are independent random samples drawn from two populations. Then the two sample Kolmogorov-Smirnov test statistic for testing H_0: the population distributions are identically same against H_1: the population distributions are different, takes a value

A) $\frac{1}{7}$ B) $\frac{2}{7}$
C) $\frac{3}{7}$ D) $\frac{3}{14}$

64. If the two random samples given in question 63 are used to test the same hypothesis H_0 against H_1 by using Wald-Wolfowitz runs test, then the value of the test criterion is equal to

A) 7 B) 14
C) 6 D) 8

65. In srswr of three units from a population of N units consider the statements:

(i) the probability of the sample containing the same unit is $\frac{1}{N^3}$

(ii) the probability of the sample containing two different units is $\frac{3(N-1)}{N^2}$

(iii) the probability of the sample containing different units is $\frac{(N-1)(N-2)}{N^2}$

Now state which of the following is correct?

A) (i) only B) (i) and (ii) only
C) (i) and (iii) only D) (ii) and (iii) only

66. A simple random sample of n units gives measurements on variables x and y with $\bar{x}$ and $\bar{y}$ as the means of observations on x and y in the sample. Then for large n, Var $\left(\hat{R}\right)$ where $\hat{R} = \frac{\bar{y}}{\bar{x}}$, is approximately equal to

A) $\frac{N-n}{Nn\bar{X}^2} \sum_{i=1}^{N} \frac{(y_i - Rx_i)^2}{N-1}, R = \frac{\bar{Y}}{\bar{X}}$

B) $\frac{N-n}{Nn\bar{Y}^2} \sum_{i=1}^{N} \frac{(y_i - Rx_i)^2}{N-1}$

C) $\frac{N-n}{Nn\bar{X}^2} \frac{\sum_{i-1}^{N} (y_i - Rx_i)^2}{R^2 (N-1)}$

D) $\frac{N-n}{Nn\bar{Y}^2} \frac{\sum_{i-1}^{N} (y_i - Rx_i)^2}{R^2 (N-1)}$

67. In a population with three stratas, the following details are available:

Strata No.	No.of units	Stratum variance
1	200	30
2	150	40
3	300	20

With the above information, in order to draw a sample of 90 units the number of sample units to be drawn from the strata number 1, 2 and 3 are

A) 28,21,41 B) 30,40,20 C) 30,30,30 D) 29,30,31

68. In simple random sampling linear regression estimate $\bar{y}_{lr} = \bar{y} + b_0 \left(\bar{X} - \bar{x}\right)$ has the variance:

A) $\frac{N-n}{Nn} \sum_{i=1}^{N} (y_i - \bar{Y})^2$

B) $\frac{N-n}{Nn} \sum_{i=1}^{N} (x_i - \bar{X})^2$

C) $\frac{N-n}{Nn} b_0 \sum_{i=1}^{N} (y_i - \bar{Y})(x_i - \bar{X})$

D) $\frac{N-n}{Nn} \sum_{i=1}^{N} \frac{\left[(y_i - \bar{Y}) - b_0 (x_i - \bar{X})\right]^2}{N-1}$

69. In order to draw a random sample of size two from a population with pdf

$$f(x) = \begin{cases} 2(1-x), 0 < x < 1 \\ 0 \quad\quad , \text{otherwise} \end{cases}$$

the random numbers in the interval [0,1] selected are 0.84 and 0.75. Then the required sample observations are

A) 0.84, 0.75

B) 0.6, 0.5

C) $\sqrt{0.84}$, $\sqrt{0.75}$

D) 0.3, 0.25

70. From a population containing 46 units a srswor of 5 units has to be selected. The two digit random numbers used for the selection are the following:
44, 95, 23, 99, 04, 90, 70, 98, 24, 77, 23. Then the units selected in the sample are:

A) 44, 03, 23, 07, 04
B) 44, 23, 04, 44, 24
C) 44, 23, 04, 24, 31
D) 44, 23, 04, 24, 23

71. Given X_1, X_2, X_3 are independent stochastic variables with common variance σ^2 and expectations given by $E(X_1) = \theta_1+2\theta_2 = E(X_3)$ and $E(X_2) = \theta_1 + \theta_3$. Consider the statements
(i) $\theta_1 + \theta_2+ \theta_3$ is estimable (ii) $3\theta_1 + 2\theta_2+ 2\theta_3$ is estimable
Now state which of the following is correct?

A) Only (i) is true
B) Only (ii) is true
C) Both (i) and (ii) are true
D) Neither (i) nor (ii) is true

72. If X_1, X_2, X_3 are variables as defined in question no.71, then the BLUE of $2\theta_1 + 2\theta_2 + \theta_3$ is

A) $2X_1 + 2X_2 + X_3$
B) $X_1 + 2X_2 + X_3$
C) $2X_1 + X_2 + X_3$
D) $\frac{1}{2}(X_1 + 2X_2 + X_3)$

73. If X_1, X_2, X_3 are variables as defined in question no.71, then the error sum of squares associated with the set-up is given by

A) $\frac{(X_3 - X_1)^2}{2}$
B) $\frac{(X_3 - X_2)^2}{2}$
C) $\frac{(X_1 - X_2)^2}{2}$
D) $\frac{(X_1 - X_2 + X_3)^2}{2}$

74. Consider the following pairs of parametric functions of a RBD
(i) $\alpha_1 - 2\alpha_2 + \alpha_3$ and $2\alpha_1 - \alpha_2 - \alpha_3$ ii) $\alpha_1 - \alpha_3$ and $\alpha_1 - 2\alpha_2 + \alpha_3$
(iii) $\alpha_2 - \alpha_3$ and $4\alpha_1 - 2\alpha_2 - 2\alpha_3$ (iv) $\alpha_1 - \alpha_3$ and $\alpha_2 - \alpha_3$
Now state which of the above pairs are orthogonal linear contrasts?

A) (i) and (iv)
B) (i) and (iii)
C) (iii) and (iv)
D) (ii) and (iii)

75. The two way classification model under consideration is given by

$Y_{ij} = \mu + \alpha_i + \beta_j + b\, X_{ij} + e_{ij}$, i = 1, 2 ,…, r and j = 1, 2, .., t where , α_i, β_j and b are parameters, X_{ij} is the value of the concomitant variable and e_{ij}, i= 1,…, r and j = 1, 2,…, t are iid error random variables with 0 means and common variance σ^2. Now state which of the following is the least-squares estimate of b?

A) $$\frac{\sum_{i,j} X_{ij}\, Y_{ij} - \frac{X_{..}\, Y_{..}}{rt}}{\sum_{i,j} X_{ij}^2 - \frac{X_{..}^2}{rt}}$$

B) $$\frac{\sum_{i,j} X_{ij}\, Y_{ij} - \sum_{i} \frac{X_{i.}\, Y_{i.}}{t}}{\sum_{i,j} X_{ij}^2 - \sum_{i} \frac{X_{i}^2}{t}}$$

C) $$\frac{\sum_{i,j} X_{ij}\, Y_{ij} - \sum_{i} \frac{X_{i.}\, Y_{i.}}{t} - \sum_{j} \frac{X_{.j}\, Y_{.j}}{r} + \frac{X_{..}\, Y_{..}}{rt}}{\sum_{i,j} X_{ij}^2 - \sum_{i} \frac{X_{i.}^2}{t} - \sum_{j} \frac{X_{.j}^2}{r} + \frac{X^2_{..}}{rt}}$$

D) $$\frac{\sum_{i,j} X_{ij}\, Y_{ij} - \sum_{j} \frac{X_{.j}\, Y_{.j}}{r}}{\sum_{i,j} X_{ij}^2 - \sum_{i,} \frac{X_{.j}^2}{t}}$$

76. In a Latin square design with t treatments, if we write R, C and E to denote the row, column and error mean squares respectively then which of the following is the efficiency of LSD relative to CRD?

A) $\frac{(t-1)R + (t-1)C + (t-1)^2 E}{(t^2-1)E}$ B) $\frac{R + C + E}{(t-1)E}$

C) $\frac{(t-1)R + (t-1)C}{(t^2-1)E}$ D) $\frac{R + C + E}{(t+1)E}$

77. The following is a design with 6 treatments arranged in 10 blocks

Block No	Treatments				Block No	Treatments		
1	1	2	5		6	2	3	4
2	1	2	6		7	2	3	5
3	1	3	4		8	2	4	6
4	1	3	6		9	3	5	6
5	1	4	5		10	4	5	6

Consider the following statements:
(i) The error degrees of freedom is 15
(ii) Standard error of difference between any two mean effects of treatments is $\frac{2\sigma^2}{5}$. Now state which of the following is correct?

A) (i) only is true B) (ii) only is true
C) Both (i) and (ii) are true D) Neither (i) nor (ii) is true

78. In a 2^4 experiment involving factors A, B, C and D, two interactions are confounded into the blocks and the principal block is {(1), bc, abd, acd}. Then the interactions confounded are
A) AC, BD, ABCD B) ABC, BCD, AD
C) BC, ABD, ACD D) AD, BC, ABCD

79. The same confounded design with principal block is {(1), bc, abd, acd} is repeated two times. Then the degrees of freedom of the error is equal to
A) 15 B) 14 C) 12 D) 10

80. Which among the following is the principal block of a 3^3 design in which AC^2 and ABC are confounded?
A) {(0,0,0), (1,2,0), (2,1,0)} B) {(0,0,0), (1,1,1), (2,2,2)}
C) {(0,0,0), (1,2,1), (1,1,2)} D) {(0,0,0), (1,2,2), (2,1,2)}

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